AP Statistics

Name: ______________________________

Central Limit Theorem

1. An insurance company has found that repair claims have a mean of \$525 with standard deviation of \$375. Because most of the claims are for minor repairs and a few are for very extensive work, the distribution is skewed to the right.

   A. A simple random sample of 60 repairs is recorded. State the mean, standard deviation, and shape of the distribution of $\bar{x}$.

   B. What is the probability that the mean repair cost for these 60 claims is greater than \$610?

   C. What is the probability that the mean repair cost for these 60 claims is between \$500 and \$600?

   D. Approximately 90% of all sample means of size 60 will be in the interval $\$525 \pm M$. Find the value of *M*.

   E. How would your answer C change if the sample size is increased to 100? Explain.

   F. The insurance company wants to choose a sample size for which $P(\bar{x} \leq 550)$ is greater than 90%. Find the smallest sample size needed for this to be true.

2. The Mars Company makes M&M's and advertises that 30% of all plain M&M's are brown. A SRS of 125 plain M&M's is drawn.

   A. Find the mean and standard deviation of $\hat{p}$, the proportion of the sample that are brown.

   B. Show that the conditions are satisfied that will ensure that the distribution of $\hat{p}$ is approximately normal.

   C. Find the probability that the proportion of brown M&M's is greater than 35%.

   D. Find the probability that the proportion of brown M&M's is between 25% and 35%.

   E. If the sample is increased to 200, how will the answer to C change? Explain.

   F. If the sample is increased to 200, how will the answer to D change? Explain.

   G. How large must the sample size be so that 90% of all sample proportion will be within 2% of the population proportion, $p = 30\%$.

   H. A sample of 125 peanut M&M's is drawn and it is found that 50 of the 125 are brown. If peanut M&M's are distributed with the same proportions as the plain M&M's, what is the probability of obtaining a sample proportion as large or larger than this one? What can be said about the proportion of brown peanut M&M's?

ANSWERS to CLT Questions:

1. A. mean = \$525, std. dev. = $\frac{\$375}{\sqrt{60}} \approx \$48.41$ The shape is approx. normal. (The CLT applies since the sample size is large.

B. $z = \frac{610-525}{\frac{375}{\sqrt{60}}} = 1.756$ P(z > 1.756) = 0.0396

C. P(500 < x < 600) = P(–0.516 < z < 1.54) = 0.635

D. z score for 95% tile is 1.645 $1.645 = \frac{X - 525}{48.41}$ $X \approx \$604.63$ $M \approx \$79.63$

E. It would increase, because a larger sample size will reduce the variation among sample means, causing more x-bars to land between \$500 and \$600.

F. z score for 90% tile ≈ 1.28 $1.28 = \frac{550-525}{\frac{325}{\sqrt{n}}}$ So n ≈ 369.5. Round UP to 370 since a larger sample size will ensure the likelihood is *greater* than or equal to \$550.

2. A. mean = .30, StdDev = $\sqrt{\frac{0.3 \bullet 0.7}{125}} \approx 0.041$

B. 0.3(125) = 37.5 and 0.7(125) = 87.5. Both are > 10, so the shape of the sampling distribution of p-hats will be approx. normal.

C. $z = \frac{0.35-0.30}{\sqrt{\frac{0.3 \bullet 0.7}{125}}} \approx 1.22$ P(z > 1.22) ≈ 0.111

D. P(0.25 < p-hat < 0.35) = P(–1.22 < z < 1.22) = 0.7775

E. It will decrease, since the variation among p-hats will decrease, causing less p-hats to fall beyond 0.35

F. It will increase since the variation among p-hats will decrease, causing more p-hats to land between 0.25 and 0.35

G. z for 95% tile ≈ 1.645 $1.645 = \frac{0.32-0.30}{\sqrt{\frac{0.3 \bullet 0.7}{n}}}$ n ≈ 1420.66, so round to 1421 to ensure the p-hats will be within 2% of p = 30%.

H. p-hat = $\frac{50}{125} = 0.4$ $\sigma_{\hat{p}} = \sqrt{\frac{0.3 \bullet 0.7}{125}} \approx 0.041$ $z = \frac{0.4-0.3}{0.041} \approx 2.43$ P(z > 2.43) ≈ 0.0075