Probability: Anticipating Patterns

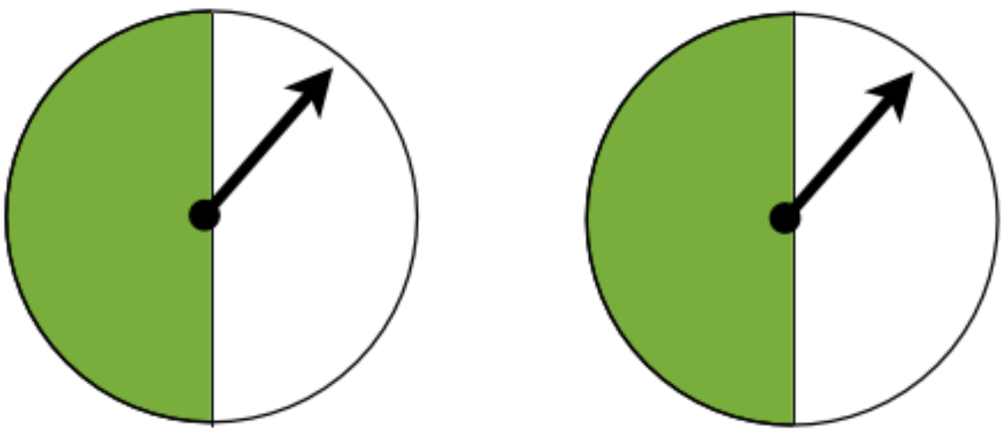
Anticipating Patterns: Exploring random phenomena using probability and simulation (20%–30%)

Probability is the tool used for anticipating what the distribution of data should look like under a given model.

A. Probability
B. Combining independent random variables
C. The normal distribution
D. Sampling distributions

Activity/Hook Ideas:

1. Two fair spinners are part of a carnival game. A player wins a prize only when both arrows land in the shaded areas after each spinner has been spun once. James is playing the game.



James thinks he has a 50-50 chance of winning. Do you agree? ***Justify your answer***.

If James plays 10 times, how many times should he expect to win? ***Explain***.

(from NCTM's *Navigating through Probability in Grades 9-12)*

2. BIG vs. small Game:

- Two players, “BIG” and “small,” play a game with a single die.
- It does not matter who rolls the single die each time (players could alternate).
- If a 5 or 6 is rolled on the die, “BIG” receives that number of points. If 1, 2, 3, or 4 is rolled, “small” gets that number of points.
- The first player to 20 points wins the game. Play 10 games and record the results.
- Is this game fair? If not, who has the advantage? Explain.

3. Casino Match War Activity:

Each student has a deck. Shuffle each deck. Students deal--simultaneously--one card at a time onto two piles. What is the probability that there will be an EXACT (suit and value) match by the time they reach the 52nd pair of dealt cards?

2012 MC Exam:

16. A complex electronic device contains three components, A, B, and C. The probabilities of failure for each component in any one year are 0.01, 0.03, and 0.04, respectively. If any one component fails, the device will fail. If the components fail independently of one another, what is the probability that the device will not fail in one year?

(A) Less than 0.01

(B) 0.078

(C) 0.080

(D) 0.922

(E) Greater than 0.99

23. A local company is interested in supporting environmentally friendly initiatives such as carpooling among employees. The company surveyed all of the 200 employees at the downtown offices. Employees responded as to whether or not they own a car and to the location of the home where they live. The results are shown in the table below.

		Location of Home			
		Downtown Area In the City	Elsewhere In the City	Outside the City	Total
Car Ownership	Yes	10	15	35	60
	No	60	55	25	140
	Total	70	70	60	200

Which of the following statements about a randomly chosen person from these 200 employees is true?

(A) If the person owns a car, he or she is more likely to live elsewhere in the city than to live in the downtown area in the city.

(B) If the person does not own a car, he or she is more likely to live outside the city than to live in the city (downtown area or elsewhere).

(C) The person is more likely to own a car if he or she lives in the city (downtown area or elsewhere) than if he or she lives outside the city.

(D) The person is more likely to live in the downtown area in the city than elsewhere in the city.

(E) The person is more likely to own a car than not to own a car.

“Traditional” Probability Problems on the AP Exam

(using Floyd Bullard’s list of priorities)

1. If 90% of the households in a certain region have answering machines and 50% have both answering machines and call waiting, what is the probability that a household chosen at random and is found to have an answering machine also has call waiting?

2. A scientist interested in right-handedness versus left-handedness and in eye color collected the following data from 1000 students:

	Right-Handedness	Left-Handedness	
Blue Eyes	210	30	240
Brown Eyes	670	90	760
	880	120	1000

   a) What is the probability that a student from this group has blue eyes?
   b) What is the probability that a student has brown eyes given that they are left handed?
   c) What is the probability that a right-handed student has blue eyes?
   d) What is the probability that a randomly selected student has blue eyes or is left-handed?
   e) What is the probability that a randomly selected student has brown eyes and is right-handed?
   f) What is the probability that a brown-eyed student is right-handed?
   g) Do eye color and handedness appear to be independent? Explain.

3. Suppose a computer company makes both laptop and desktop computers and has manufacturing plants in three states. 50% of their computers are manufactured in California and 85% of these are desktops, 30% of computers are manufactured in Washington, and 40% of these are laptops, and the rest are manufactured in Oregon, 40% of which are desktops. All computers are first shipped to a distribution center in Nebraska before being sent out to stores.
   a. If you picked a computer at random from the Nebraska distribution center, what is the probability that it is a laptop?
   b. If you picked a computer at random from the Nebraska distribution center and it was a laptop, what is the probability that it was manufactured in Washington?
   c. If you picked a computer at random from the Nebraska distribution center, what is the probability it was a desktop computer made in Oregon?

4. When rolling two dice, what is the probability that the sum is 7 given that one die is a 5?

5. Draw one card from a standard deck of 52 playing cards. Let A = “the card drawn is a spade.” Let B= “the card drawn is a queen.” Let C= “the card drawn is a 2, 3, 4, or 5.”
   a. Are A and B independent?
   b. Are B and C independent?
   c. Are any two events disjoint?

Strange Dice Problem: A game is played with 2 strange dice. The six faces of Die A show a 1 and five 3's. Die B has four 2's and two 6's.

a. Create a probability model for the total you get when you roll both dice.

b. Find the mean and standard deviation.

Flip A Fair Coin:... (or simulate)

Think about the NUMBER of heads vs. *expected* number of heads…

Think about the PERCENT of heads vs. *expected* percent of heads…

What happens in the long run?

The Coinless Coke Machine and The Law of Large Numbers (activity)

Tricky coin problem:
You've already seen 14/20 heads. How many heads should you expect after 100 total flips?

Binomial Probability

Conditions: 1.

2.

3.

4.

24% of plain M&M's are blue; n = 6; Let X = "# of blue M&M's.

1. WITPO getting exactly 4 blue out of a random sample of 6 M&M's?

2. P(X > 2)

3. P(X = 4 OR X = 5)?

4. P(no more than 3 are blue)?

5. P(at least one is blue)?

6. Create the probability distribution table.

7. Calculate the mean and standard deviation. mean = _________ SD = __________

Hat Game Example (in statistics, $3X \neq X + X + X$)

Consider a game of chance in which the player draws prizes from a hat. The prizes in the hat are labeled \$1, \$2, and \$4 with the following probability distribution:

X	\$1	\$2	\$4
$P(X)$	0.5	0.4	0.1

Find each of the following

$\mu_X =$ ________ $\sigma_X =$ ________ $\sigma_X^2 =$ ________

Case 1--"*For the next 20 minutes, the prize money is tripled.*"

$Y = 3X$	\$3	\$6	\$12
$P(Y)$			

Find each of the following:

$\mu_Y =$ ________ $\sigma_Y =$ ________ $\sigma_Y^2 =$ ________

Case 2--"*For the next 20 minutes, play 3 times for the price of one*"

Z	\$3	\$4	\$5	\$6	\$7	\$8	\$9	\$10	\$12
$P(Z)$									

Find each of the following:

$\mu_Z =$ ________ $\sigma_Z =$ ________ $\sigma_Z^2 =$ ________

There are formulas for these situations, but it's more important to understand ***conceptually***…

$\mu_{X+X+X} = \mu_X + \mu_X + \mu_X$ $(\sigma_{X+X+X} \neq \sigma_{3X})$ $\sigma_{X+X+X} = \sqrt{\sigma_X^2 + \sigma_X^2 + \sigma_X^2}$

$\mu_{3X} = 3\mu_X$ $\sigma_{3X} = 3\sigma_X$

AP Statistics: Random Variables

Developing the Rules for +/–   Name____

Let X be a random variable with equally likely outcomes {2, 4, 6, 8}

Let Y be a random variable with equally likely outcomes {1, 3, 5}

E(X) = ____________   E(Y) = ______________

SD(X) = ____________   SD(Y) = ______________

VAR(X)= ____________   VAR(Y)= _______________

Now find the set of outcomes for X + Y: {____}

Store these values in a list and calculate the mean, SD and variance.

E(X + Y) = ___________   SD(X + Y) = ____________   VAR(X + Y) = _________

Find the set of outcomes for X – Y: {____}

Store these values in a list and calculate the mean, SD and variance.

E(X – Y) = ____________   SD(X – Y) = _____________   VAR(X – Y) = __________

You should be able to verify that for the sum or difference of ***independent*** random variables,

____________________ ________.

Practice Problem:

The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces (45 pounds) and a standard deviation of 4 ounces. The bars are shipped 10 in a box to the retailers. The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces. The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces and a standard deviation of

(A) $\sqrt{12}$ ounces
(B) $\sqrt{80}$ ounces
(C) $\sqrt{224}$ ounces
(D) 48 ounces
(E) $\sqrt{1{,}664}$ ounces

Random Variables Practice

(from SYM 4e) NAME____

<u>Show all work and calculations in the space provided.</u>

1. Mr. Starnes likes sugar in his hot tea. From experience, he needs between 8.5 and 9 grams of sugar in a cup of tea for the drink to taste right. While making his tea one morning, Mr. Starnes adds four randomly selected packets of sugar. Suppose the amount of sugar in these packets follows a Normal distribution with mean 2.17 grams and standard deviation 0.08 grams.

   What is the probability that Mr. Starnes's tea tastes right?

2. The diameter C of a randomly selected large drink cup at a fast-food restaurant follows a Normal distribution with a mean of 3.96 inches and a standard deviation of 0.01 inches. The diameter L of a randomly selected large lid at this restaurant follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. For a lid to fit on a cup, the value of L has to be bigger than the value of C, but not by more than 0.06 inches.

   What is the probability that a randomly selected lid will fit on a randomly chosen large drink cup?

AP Statistics

Name: ___________________________

Roll until "doubles"

A game of chance is played in which two dice are rolled until "doubles" appear. A trial consists of a sequence of rolls terminating with a roll of "doubles". Let X = the roll on which doubles first appears. (Rolls of {2, 3}, {4, 1}, {2, 6} and {3, 3} would give $x = 4$)

Discuss predicted answers to the following questions.

1. On which roll is it more likely to roll doubles? Justify your answer.

2. By which roll do you typically roll doubles?

3. Describe the shape, center and spread for this distribution.

4. Locate the mean and the median for this distribution. Which is larger? Why?

5. What about this game: If you can roll the dice 6 times <u>*without*</u> rolling "doubles", I will give you $1. However, if "doubles" are rolled on the first through sixth roll, you pay me $1…. Who wants to play?

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

   (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

   (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

   (c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

   Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Homework: Score the three student responses to Part (b) below:

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes, there is reason to doubt the managers claim that the 3 people were selected at random because the probability for him to randomly select them is 1.2%. This means that it is very unlikely that he selected them at random.

Student "T"   Score for this part: E P I

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes there is reason to doubt the manager's claim that 3 people were selected at random since there is only a 1.19% that all 3 women will be selected for the trip which is very unlikely. since all 3 women were chosen, it shows that the selection was not random.

Student "F"   Score for this part: E P I

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes there is reason to doubt the manager's claim because the probability is so low.

Student "M"   Score for this part: E P I

Answers to "Traditional" Probability Problems: (Floyd Bullard's list)

1. 5/9
2. a) 240/1000
   b) 90/120
   c) 210/880
   d) 330/1000
   e) 670/1000
   f) 670/760
   g) Yes. The ratio of blue eyed:brown eyed left-handers is 1:3, and the ratio among righties is 21/67, nearly the same. (Other similar answers are possible, showing nearly equal ratios.)
3. a) .315
   b) .12/.315
   c) .08
4. 2/11
5. a) Yes, because P(spade|queen) = P(spade) = ¼
   b) No, because P(queen|2,3,4, or 5) = 0 and P(queen) = 1/13
   c) Yes, B and C are disjoint since both events cannot happen simultaneously (a card cannot be a queen and a 2, 3, 4, or 5 at the same time.

Mr. Starnes's Tea and Lid problems:

1. Mu = 8.68, sigma = sqrt(0.0256) = 0.16
   z = 8.5–8.68/0.16 = –1.13 and z = 9–8.68/0.16 = 2.00
   P(–1.13 ≤ z ≤ 2.00) = 0.8480 ≈ 85% chance

2. Mu = 3.98–3.96 = 0.02, sigma = sqrt(0.0005) = 0.0224
   z = 0–0.02/0.0224 = –.89 and z = 0.06–0.02/0.0224 = 1.79
   P(–0.89 ≤ z ≤ 1.79) = 0.7766 ≈ 78%

Random Variable Problem: Attila Barbells:
Answer is (c): √224

Steely Dan problem: ≈ 28.6%

Homework: I, P, E