

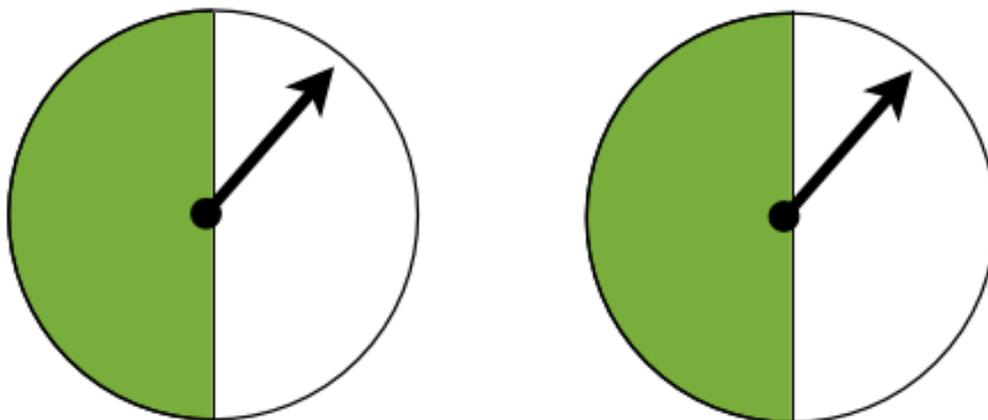
# Probability: Anticipating Patterns

Anticipating Patterns: Exploring random phenomena using probability and simulation (20%–30%)  
*Probability is the tool used for anticipating what the distribution of data should look like under a given model.*

- A. Probability C. The normal distribution
- D. Sampling distributions
- B. Combining independent random variables

## Activity/Hook Ideas:

1. Two fair spinners are part of a carnival game. A player wins a prize only when both arrows land in the shaded areas after each spinner has been spun once. James is playing the game.



James thinks he has a 50-50 chance of winning. Do you agree? **Explain.**

If James plays 10 times, how many times should he expect to win? **Explain.**

(from NCTM's *Navigating through Probability in Grades 9-12*)

## 2. BIG vs. small Game:

- Two players, "BIG" and "small," play a game with a single die.
- It does not matter who rolls the single die each time (players could alternate).
- If a 5 or 6 is rolled on the die, "BIG" receives that number of points. If 1, 2, 3, or 4 is rolled, "small" gets that number of points.
- The first player to 20 points wins the game. Play 10 games and record the results.
- Is this game fair? If not, who has the advantage? Explain.

## 3. Casino Match War Activity:

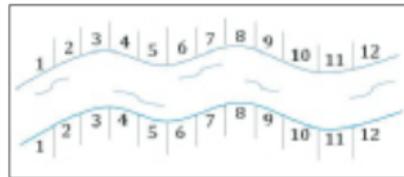
Each student has a deck. Shuffle each deck. Students deal--simultaneously--one card at a time onto two piles. What is the probability that there will be an EXACT (suit and value) match by the time they reach the 52nd pair of dealt cards?

## 4. Probability Bingo (see noblestatman.com)

**Probability Bingo**  
(From Brian Mehmed, 2011 WV APSI)

Each of two die has *colored* faces, 3 green, 2 blue and 1 red. The two dice will be rolled. The outcome will be considered to be one "bingo call." If you have this outcome on your bingo card, mark it off. The winner will be the student who gets a bingo card completely marked off (all 25 squares). Mark each square on your bingo card (use BG for "blue green," BB for "blue blue," etc.) so that you have the best chance of winning.


## 5. "Playing the River Crossing Game," *Mathematics Teacher*: Vol. 105, No. 4 November 2011, p. 320



Each team gets 12 boats, and can put any number of them in any of 12 docks.

Teams alternate rolling a pair of dice. If your team has a boat in that dock, then one more boat has crossed. Team that gets all 12 boats across first is the winner.

6. Foam dice vs. casino dice vs. Sicherman or other "trick" dice. Roll dice to create a probability table (model) for each type of dice. Analyze for "fairness."

## 7. Greedy Pig game (and strategy discussion from BVD teacher resources)

"My first iPod loved Steely Dan. So do I. But not as much as my iPod did.... I didn't keep track of every song that played every time I shuffled my tunes, but after a while I would keep a sharp ear out for what I came to call the LTBSD (Length of Time Before Steely Dan) Factor. The LTBSD Factor was always perplexingly short."

-Stephen Levy, writing in the Guardian about the perception of randomness, or the lack thereof, on an iPod set to shuffle.

**Assume you have 3,000 songs on your mp3 player, 50 of which are Steely Dan songs. How likely is it that you will hear at least one Steely Dan song in the first 20 songs after a "shuffle?"**

-from *Statistics, Learning from Data*, pp. 342-343

## “Traditional” Probability Problems on the AP Exam

(using Floyd Bullard’s list of priorities)

1. If 90% of the households in a certain region have answering machines and 50% have both answering machines and call waiting, what is the probability that a household chosen at random and is found to have an answering machine also has call waiting?

2. A scientist interested in right-handedness versus left-handedness and in eye color collected the following data from 1000 students:

	Right-Handedness	Left-Handedness	
Blue Eyes	210	30	240
Brown Eyes	670	90	760
	880	120	1000

- a) What is the probability that a student from this group has blue eyes?
- b) What is the probability that a student has brown eyes given that they are left handed?
- c) What is the probability that a right-handed student has blue eyes?
- d) What is the probability that a randomly selected student has blue eyes or is left-handed?
- e) What is the probability that a randomly selected student has brown eyes and is right-handed?
- f) What is the probability that a brown-eyed student is right-handed?
- g) Do eye color and handedness appear to be independent? Explain.

3. Suppose a computer company makes both laptop and desktop computers and has manufacturing plants in three states. 50% of their computers are manufactured in California and 85% of these are desktops, 30% of computers are manufactured in Washington, and 40% of these are laptops, and the rest are manufactured in Oregon, 40% of which are desktops. All computers are first shipped to a distribution center in Nebraska before being sent out to stores.
- If you picked a computer at random from the Nebraska distribution center, what is the probability that it is a laptop?
  - If you picked a computer at random from the Nebraska distribution center and it was a laptop, what is the probability that it was manufactured in Washington?
  - If you picked a computer at random from the Nebraska distribution center, what is the probability it was a desktop computer made in Oregon?
4. When rolling two dice, what is the probability that the sum is 7 given that one die is a 5?
5. Draw one card from a standard deck of 52 playing cards. Let  $A$  = “the card drawn is a spade.” Let  $B$  = “the card drawn is a queen.” Let  $C$  = “the card drawn is a 2, 3, 4, or 5.”
- Are  $A$  and  $B$  independent?
  - Are  $B$  and  $C$  independent?
  - Are any two events disjoint?



# Binomial Probability

- Conditions:
- 1.
  - 2.
  - 3.
  - 4.

**24% of plain M&M's are blue;  $n = 6$ ; Let  $X =$  “# of blue M&M's.”**

1. WITPO getting exactly 4 blue out of a random sample of 6 M&M's?
2.  $P(X > 2)$
3.  $P(X = 4 \text{ OR } X = 5)$ ?
4.  $P(\text{no more than 3 are blue})$ ?
5.  $P(\text{at least one is blue})$ ?
6. Create the probability distribution table.
7. Calculate the mean and standard deviation. mean = \_\_\_\_\_ SD = \_\_\_\_\_

## Hat Game Example (in statistics, $3X \neq X + X + X$ )

Consider a game of chance in which the player draws prizes from a hat. The prizes in the hat are labeled \$1, \$2, and \$4 with the following probability distribution:

$X$	\$1	\$2	\$4
$P(X)$	0.5	0.4	0.1

Find each of the following

$$\mu_X = \underline{\hspace{2cm}} \qquad \sigma_X = \underline{\hspace{2cm}} \qquad \sigma_X^2 = \underline{\hspace{2cm}}$$

Case 1-- "For the next 20 minutes, the prize money is tripled."

$Y = 3X$	\$3	\$6	\$12
$P(Y)$			

Find each of the following:

$$\mu_Y = \underline{\hspace{2cm}} \qquad \sigma_Y = \underline{\hspace{2cm}} \qquad \sigma_Y^2 = \underline{\hspace{2cm}}$$

Case 2-- "For the next 20 minutes, play 3 times for the price of one"

$Z$	\$3	\$4	\$5	\$6	\$7	\$8	\$9	\$10	\$12
$P(Z)$									

Find each of the following:

$$\mu_Z = \underline{\hspace{2cm}} \qquad \sigma_Z = \underline{\hspace{2cm}} \qquad \sigma_Z^2 = \underline{\hspace{2cm}}$$

Here are the geeky math formulas...but it's more important to understand conceptually what's going on...

$$\mu_{nX} = \underline{\hspace{2cm}} \qquad \sigma_{nX} = \underline{\hspace{2cm}} \qquad \sigma_{nX}^2 = \underline{\hspace{2cm}}$$

and

$$\mu_{X_1+X_2+\dots+X_n} = \underline{\hspace{2cm}} \qquad \sigma_{X_1+X_2+\dots+X_n} = \underline{\hspace{2cm}} \qquad \sigma_{X_1+X_2+\dots+X_n}^2 = \underline{\hspace{2cm}}$$

# AP Statistics: Random Variables

Developing the Rules for +/-

Name \_\_\_\_\_

Let  $X$  be a random variable with equally likely outcomes  $\{2, 4, 6, 8\}$

Let  $Y$  be a random variable with equally likely outcomes  $\{1, 3, 5\}$

$$E(X) = \underline{\hspace{2cm}}$$

$$E(Y) = \underline{\hspace{2cm}}$$

$$SD(X) = \underline{\hspace{2cm}}$$

$$SD(Y) = \underline{\hspace{2cm}}$$

$$VAR(X) = \underline{\hspace{2cm}}$$

$$VAR(Y) = \underline{\hspace{2cm}}$$

Now find the set of outcomes for  $X + Y$ :  $\{\underline{\hspace{10cm}}\}$

Store these values in a list and calculate the mean, SD and variance.

$$E(X + Y) = \underline{\hspace{2cm}}$$

$$SD(X + Y) = \underline{\hspace{2cm}}$$

$$VAR(X + Y) = \underline{\hspace{2cm}}$$

Find the set of outcomes for  $X - Y$ :  $\{\underline{\hspace{10cm}}\}$

Store these values in a list and calculate the mean, SD and variance.

$$E(X - Y) = \underline{\hspace{2cm}}$$

$$SD(X - Y) = \underline{\hspace{2cm}}$$

$$VAR(X - Y) = \underline{\hspace{2cm}}$$

You should be able to verify that for the sum or difference of *independent* random variables,

\_\_\_\_\_.

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## Practice Problem:

The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces (45 pounds) and a standard deviation of 4 ounces. The bars are shipped 10 in a box to the retailers. The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces. The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces and a standard deviation of

- (A)  $\sqrt{12}$  ounces
- (B)  $\sqrt{80}$  ounces
- (C)  $\sqrt{224}$  ounces
- (D) 48 ounces
- (E)  $\sqrt{1,664}$  ounces

# Random Variables Practice

(from SYM 4e)

NAME \_\_\_\_\_

**Show all work and calculations in the space provided.**

1. Mr. Starnes likes sugar in his hot tea. From experience, he needs between 8.5 and 9 grams of sugar in a cup of tea for the drink to taste right. While making his tea one morning, Mr. Starnes adds four randomly selected packets of sugar. Suppose the amount of sugar in these packets follows a Normal distribution with mean 2.17 grams and standard deviation 0.08 grams.

What is the probability that Mr. Starnes's tea tastes right?

2. The diameter  $C$  of a randomly selected large drink cup at a fast-food restaurant follows a Normal distribution with a mean of 3.96 inches and a standard deviation of 0.01 inches. The diameter  $L$  of a randomly selected large lid at this restaurant follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. For a lid to fit on a cup, the value of  $L$  has to be bigger than the value of  $C$ , but not by more than 0.06 inches.

What is the probability that a randomly selected lid will fit on a randomly chosen large drink cup?

# AP Statistics

Name: \_\_\_\_\_

Roll until “doubles”

A game of chance is played in which two dice are rolled until “doubles” appear. A trial consists of a sequence of rolls terminating with a roll of “doubles”. Let  $X$  = the roll on which doubles first appears. (Rolls of  $\{2, 3\}$ ,  $\{4, 1\}$ ,  $\{2, 6\}$  and  $\{3, 3\}$  would give  $x = 4$ )

**Discuss predicted answers to the following questions.**

1. On which roll is it more likely to roll doubles? Justify your answer.
2. By which roll do you typically roll doubles?
3. Describe the shape, center and spread for this distribution.
4. Locate the mean and the median for this distribution. Which is larger? Why?
5. What about this game: If you can roll the dice 6 times without rolling “doubles”, I will give you \$1. However, if “doubles” are rolled on the first through sixth roll, you pay me \$1.... Who wants to play?

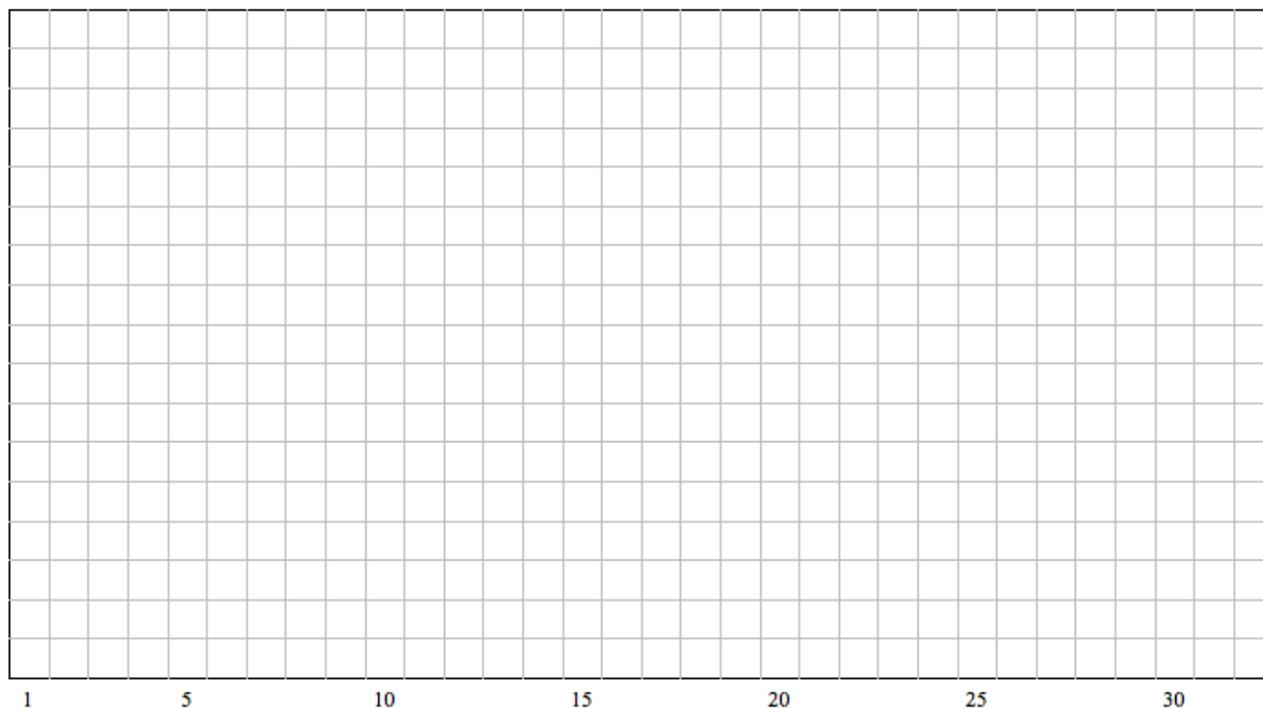
"Rolling until doubles" can be simulated on the TI 83 or TI 84 calculator using the `randInt(1,6,2)` command. This will select two integers at random from 1 to 6 inclusively.

For example, if your first trial was {2, 3}, {3, 5}, {6, 2}, {1, 4} and {4, 4}, you rolled doubles on the 5<sup>th</sup> roll. So  $x = 5$  for that trial.

Conduct several trials of the game.

Trial	# Rolls until doubles						
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

Construct a histogram of the class results on the grid below.



**Extra Credit:** What is the difference between  $X$  and  $x$ ?

## Homework: Score the three student responses to Part (b) below:

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.
- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.
- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.
- (c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Student "T"      Score for this part:   E   P   I

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes, there is reason to doubt the manager's claim that the 3 people were selected at random because the probability for him to randomly select them is 1.2%. This means that it is very unlikely that he selected them at random.

Student "F"

Score for this part: E P I

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes there is reason to doubt the manager's claim that 3 people were selected at random since there is only a 1.19% that all 3 women will be selected for the trip which is very unlikely, since all 3 women were chosen, it shows that the selection was not random.

Student "M"

Score for this part: E P I

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes there is reason to doubt the manager's claim because the probability is so low.

## ANSWERS to Basic Probability Problems

1.  $1 - .4 = .6$  or 60%
2.  $.3 + .1 = .4$  or 40%
3.  $.3 \cdot .3 = .09$  or 9%
4.  $.9 \cdot .9 \cdot .9 = .729$  or 72.9%
5.  $1 - (.6 \cdot .6 \cdot .6) = 1 - .216 = .784$  or 78.4%
6.  $.7 \cdot .7 \cdot .7 \cdot .3 = .1029$  or 10.29%
7. Using the binomial formula:  
$$\binom{18}{4} \cdot .3^4 \cdot .7^{14} \approx .1681$$
 or 16.81%
8.  $P(X \geq 6) = 1 - \text{binomcdf}(12, .10, 5) = 5.4 \times 10^{-4} \approx 0\%$
9.  $P(X \leq 8) = \text{binomcdf}(16, .30, 8) \approx .9743$  or 97.43%

## Answers to More Probability Problems: (using Floyd Bullard's list)

1.  $5/9$
2. a)  $240/1000$   
b)  $90/120$   
c)  $210/880$   
d)  $330/1000$   
e)  $670/1000$   
f)  $670/760$   
g) Yes. The ratio of blue eyed:brown eyed left-handers is 1:3, and the ratio among righties is  $21/67$ , nearly the same. (Other similar answers are possible, showing nearly equal ratios.)
3. a)  $.315$   
b)  $.12/.315$   
c)  $.08$
4.  $2/11$
5. a) Yes, because  $P(\text{spade}|\text{queen}) = P(\text{spade}) = 1/4$   
b) No, because  $P(\text{queen}|2,3,4, \text{ or } 5) = 0$  and  $P(\text{queen}) = 1/13$   
c) Yes, B and C are disjoint since both events cannot happen simultaneously (a card cannot be a queen and a 2, 3, 4, or 5 at the same time.)

## Mr. Starnes's Tea and Lid problems:

1.  $\mu = 8.68$ ,  $\sigma = \sqrt{0.0256} = 0.16$   
 $z = 8.5 - 8.68/0.16 = -1.13$  and  $z = 9 - 8.68/0.16 = 2.00$   
 $P(-1.13 \leq z \leq 2.00) = 0.8480 \approx 85\%$  chance
2.  $\mu = 3.98 - 3.96 = 0.02$ ,  $\sigma = \sqrt{0.0005} = 0.0224$   
 $z = 0 - 0.02/0.0224 = -0.89$  and  $z = 0.06 - 0.02/0.0224 = 1.79$   
 $P(-0.89 \leq z \leq 1.79) = 0.7766 \approx 78\%$

## Random Variable Problem: Attila Barbells:

Answer is (c):  $\sqrt{224}$

Steely Dan problem:  $\approx 28.6\%$

Homework: I, P, E