

# AP Statistics Probability FRAPPY!

NAME \_\_\_\_\_

Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

## **FRAPPY Follow-up:**

1. Describe the "Intent of the Question" from your perspective. In other words, "What do you think this question was getting at? What statistical concepts or abilities are they asking you to display?"

a)

b)

c)

2. (answer at the end of the activity) Describe below weak areas revealed by this question. What will you need to improve upon in order to be better prepared for the AP Statistics Exam?

## Student E:

- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$$(3/9)(2/8)(1/7) = .012$$

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

There is a small ~~chance~~ chance that these results would occur if the selection was random, therefore there is reason to doubt the manager's claim.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

The proposed process does not simulate the selection correctly because it assumes the probability ~~that~~ that a woman will be selected is  $1/3$  and that the trials are independent of each other, both of which are not true.

## Student A:

- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$$P = \frac{3}{9} \left( \frac{2}{9} \right) \left( \frac{1}{9} \right) = .00823$$

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes, there is reason to doubt the manager's claim because w/ a probability of .00823, it is highly unlikely that out of a pick of 3, all 3 women were chosen. Even though it is possible, one would expect doubt to arise w/ a .00823 chance of all 3 women being picked randomly.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

No, the proposed simulation does not simulate the random selection because each time you roll a die you have a  $\frac{2}{6}$  or  $\frac{1}{3}$  chance of rolling a 5 or a 6, which the proposed process says represents women. However, in the actual random selection of men/women the probability of choosing a woman goes down after one has already been selected because the # of women to choose from has now decreased. So instead of a  $\frac{3}{9}$  chance you go down to a  $\frac{2}{9}$  chance once one woman has been selected, diminishing the odds of picking another woman.

## Student B:

- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$$\begin{aligned} \text{Probability of selecting 1 woman} &= \frac{3}{9} = \frac{1}{3} \\ 2 \text{ women} &= \frac{2}{8} \\ 3 \text{ women} &= \frac{1}{7} \\ \left(\frac{1}{3}\right) \left(\frac{2}{8}\right) \left(\frac{1}{7}\right) &= \frac{1}{84} = .0119 \end{aligned}$$

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

There is reason to doubt the claim because there is a 1% chance that 3 women would be selected and this is highly unlikely.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

YES. In order to correctly perform a simulation, the probabilities of obtaining a random number should be the same as attaining the corresponding action in real life. In this case, the probability of rolling a 5 or 6 is the same probability of selecting a woman at random. The probability ~~is~~ is  $\frac{1}{3}$ . The probability of rolling a 1, 2, 3, or 4 is the same as selecting a man;  $\frac{2}{3}$ .

## Student C:

- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$$P(3 \text{ women selected}) = \left(\frac{3}{9}\right)\left(\frac{2}{8}\right)\left(\frac{1}{7}\right) \approx 0.0119 \text{ or } \approx 1.2\% \text{ chance}$$

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes, because there is only a 1.2% chance that the 3 women were chosen at random, and we are generally suspicious of anything under a 5% chance of occurring randomly.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

No because while the probability of a man being chosen <sup>for one slot</sup> is  $\frac{6}{9}$  which is equivalent to  $\frac{4}{6}$ , and the probability of a woman being chosen for one slot is  $\frac{3}{9}$  which is equivalent to  $\frac{2}{6}$ , this is only true for the first selection. Once an individual is chosen to attend the convention, the pool of remaining employees decreases to 8 and then 7.

## Student D:

- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

$$P(M) = 6/9 = 0.6667$$

$$P(W) = 3/9 = 0.3333$$

$$\binom{3}{9} \binom{2}{8} \binom{1}{7} = \boxed{0.01190}$$

w/o replacement

- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

Yes there is reason to doubt the manager's claim that 3 people were selected at random since there is only a 1.19% that all 3 women will be selected for the trip which is very unlikely. Since all 3 women were chosen, it shows that the selection was not random.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Yes the proposed process correctly simulates the random selection since rolling a 5 or 6 represents selecting a woman, that proportion equals the proportion of 3 women out of a group of 9. ( $\frac{2}{6} = \frac{1}{3} = \frac{3}{9}$ ) Likewise for men, rolling 1-4 correctly shows the proportion of selecting a man from a group of 9. ( $\frac{4}{6} = \frac{2}{3} = \frac{6}{9}$ ) Since the proportions match the group, this process correctly simulates the random selection.

## Student E: Score = 4

Part (a): Correct answer and work is shown. Essentially correct.

Part (b): In part (b) the student correctly states that “there is reason to doubt the manager’s claim” and supports this assertion by saying that “There is a small chance that these results would occur if the selection was random,” which is appropriate for such a small probability of 0.012. The communication is very strong, using the conditional tense (“would occur”), and conditioning the probability upon the claimed random selection (“if the selection was random”). Part (b) was scored as essentially correct.

Part (c): The student correctly states that “[t]he proposed process does not simulate the selection correctly.” The student then correctly addresses the independence of the outcomes of the three dice by saying that “the trials are independent of each other,” as well as by indicating that the proposed process “assumes the probability that a woman will be selected is  $1/3$ .” The response would have been stronger had the student indicated the probability was the same for all three selections, but the overall communication is clear enough that the phrase is implied. Finally, the student addresses the dependence of the genders of the selected convention attendees succinctly but clearly by concluding, “both of which are not true.” The response in part (c) was scored as essentially correct.

## Student A: Score = 3

In part (a) the computation shows a recognition that after each woman is selected, the number of remaining women decreases. This is indicated by the numerators of the three component fractions, which decrease from 3 to 2 to 1. However, the student fails to recognize that the total number of people remaining also should decrease from 9 to 8 to 7. Part (a) was scored as partially correct. In part (b) the student correctly states that “there is reason to doubt the manager’s claim,” and correctly supports this statement by saying that “it is *highly unlikely* that out of a pick of 3, all 3 women were chosen” (italics added). The response would have been stronger had the student used the conditional tense of *would have been chosen* rather than the past tense *were chosen*, since the probability requested in part (a) is not that of a past event, but of a conditional event—the selection of 3 women supposing that the selection is done at random. But that weakness in communication is sufficiently minor that it does not alter the score. Part (b) was scored as essentially correct. In part (c) the student correctly states that “the proposed simulation does not simulate the random selection.” The student correctly addresses the independence of the dice outcomes by saying that “*each time* you roll a die you have a  $2/6$  or  $1/3$  chance of rolling a 5 or a 6” (italics added), which implies independence. The student contrasts this with the dependence of the genders of the selected convention attendees by stating that “in the actual random selection of men & women the probability of choosing a woman goes down after one has already been selected”, which implies dependence. The overall communication in part (c) is quite strong—for example, through its use of the contrasting word “[h]owever.” Part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

## Student B: Score = 2

In part (a) the student gives the correct probability of 0.0119 and shows how it was computed. The response includes a communication error, in that it states “2 women =  $\frac{2}{8}$ ” and “3 women =  $\frac{1}{7}$ ” rather than the more accurate  $P(\text{2nd woman}) = \frac{2}{8}$  and  $P(\text{3rd woman}) = \frac{1}{7}$ . However, the error is minor and does not affect the score in part (a). Part (a) was scored as essentially correct. In part (b) the response correctly states that “[t]here is reason to doubt the claim” and supports this statement by saying that “this is highly unlikely,” which is appropriate for the small probability of 1 percent. The response is strengthened by the use of the conditional tense (“would be selected”). The response would have been stronger if the student had indicated that the selection had actually been random, but the communication is strong enough that this is implicit. Part (b) was scored as essentially correct. In part (c) the student incorrectly states that the proposed process successfully simulates the selection of 3 women from a group of 9 people consisting of 6 men and 3 women. Although the student recognizes that the chance of selecting one woman and the chance of rolling a 5 or a 6 on one die are equal  $\left(\frac{1}{3}\right)$ , the student fails to recognize that there is dependence between the genders of the three selected convention attendees and does not contrast that with the independence of the dice outcomes. Part (c) was scored as incorrect. Because two parts were scored as essentially correct and one part was scored as incorrect, the response earned a score of 2.

## Student C: Score = 2

Part (a): Essentially correct

Part (b): “Yes” is the correct response with “only” 1.2%. The “chance that the 3 women were chosen at random” is an incorrect interpretation of the probability calculated in part (a). The correct interpretation of 1.2% is “the probability that a random selection process would have chosen three women.” Partially correct.

Part (c): “No” is the correct answer, but the explanation only describes the dependence of the genders in the selection process. There is no contrast to the independence of the dice rolls. This leaves the comparison entirely implicit. Partially correct. EPP = 2

## Student D: Score = 2

Part (a): Essentially correct

Part (b): “Yes” is correct, and the explanation is sufficient and correct. However, the student ends their answer with “it shows that the selection was not made at random.” This is too strong. There is insufficient evidence to PROVE that the manager did not make the selection at random. Partially correct.

Part (c): Incorrectly states “yes,” and does not address the independence of the die or the dependence of the genders.