AP Statistics FR Questions: Probability

1. 1999 #5: Rolling Dice

Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.

(a) Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

(b) Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive ach time he or she wins in order for this to be a fair game? Explain how you found your answer.

(A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens taken in the long run.)

2. 2006B #3: Golf Balls

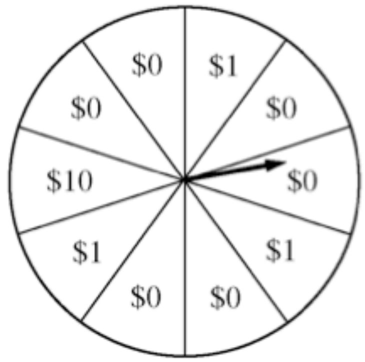
Golf balls must meet a set of five standards in order to be used in professional tournaments. One of these standards is distance traveled. When a ball is hit by a mechanical device, Iron Byron, with a 10-degree angle of launch, a backspin of 42 revolutions per second, and a ball velocity of 235 feet per second, the distance the ball travels may not exceed 291.2 yards. Manufacturers want to develop balls that will travel as close to the 291.2 yards as possible without exceeding that distance. A particular manufacturer has determined that the distances traveled for the balls it produces are normally distributed with a standard deviation of 2.8 yards. This manufacturer has a new process that allows it to set the mean distance the ball will travel.

(a) If the manufacturer sets the mean distance traveled to be equal to 288 yards, what is the probability that a ball that is randomly selected for testing will travel too far?

(b) Assume the mean distance traveled is 288 yards and that five balls are independently tested. What is the probability that at least one of the five balls will exceed the maximum distance of 291.2 yards?

3. 2012 #2: Charity Fundraiser

A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	–$8
$P(x)$			

(b) What is the expected value of the net contribution to the charity for one play of the game?

(c) The charity would like to receive a net contribution of $500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least $500 ?

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least $500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are $700 and $92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least $500 in 1,000 plays of the game.

4. 2005B: Concert Tickets

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

c	0	1	2	3
$p(c)$	0.4	0.3	0.2	0.1

(a) Compute the mean and the standard deviation of C.

(b) Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the numbers of child tickets and adult tickets purchased are independent random variables. Compute the mean and the standard deviation of the total number of adult and child tickets purchased by a single customer.

(c) Suppose each child ticket costs \$15 and each adult ticket costs \$25. Compute the mean and the standard deviation of the total amount spent per purchase.

5. 2006 #3: Earth Depth

The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

(a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?

(b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that at least one of these measurements will be negative?

(c) What is the probability that the mean of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?

6. 2002 #3: Runners

There are 4 runners on the New High School team. The team is planning to participate in a race in which each runner runs a mile. The team time is the sum of the individual times for the 4 runners. Assume that the individual times of the 4 runners are all independent of each other. The individual times, in minutes, of the runners in similar races are approximately normally distributed with the following means and standard deviations.

	Mean	Standard Deviation
Runner 1	4.9	0.15
Runner 2	4.7	0.16
Runner 3	4.5	0.14
Runner 4	4.8	0.15

(a) Runner 3 thinks that he can run a mile in less than 4.2 minutes in the next race. Is this likely to happen? Explain.

(b) The distribution of possible team times is approximately normal. What are the mean and standard deviation of this distribution?

(c) Suppose the team's best time to date is 18.4 minutes. What is the probability that the team will beat its own best time in the next race?

ANSWERS:

AP Statistics FR Questions: Probability

1. Rolling Dice (1999, #5):

Possible Outcomes

Die A	Die B	Winner	Prob
9	3	A	(2/3)(2/3) = 4/9
9	11	B	(2/3)(1/3) = 2/9
0	3	B	(1/3)(2/3) = 2/9
0	11	B	(1/3)(1/3) = 1/9

OR

	DIE A						
DIE B		0	0	9	9	9	9
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	11	B	B	B	B	B	B
	11	B	B	B	B	B	B

Winner	Prob
A	16/36 = 4/9
B	20/36 = 5/9

a. Choose die B, because the probability of winning is higher (5/9 compared to 4/9 for die A)
b. Let X be the number of tokens the player using die B should receive. For the game to be fair, we need

   $45(4/9) = X(5/9)$

   Solving this equation for X gives X = 36. Player B should receive 36 tokens.

2. Golf Balls (2006B #3)

Part (a):

Let D represent the distance a randomly selected ball travels. Since D is normally distributed with a mean of 288 yards and a standard deviation of 2.8 yards, we find

$$P(D > 291.2) = P\left(Z > \frac{291.2 - 288}{2.8}\right) > P(Z > 1.14) = 1 - 0.8729 = 0.1271.$$

Part (b):

$$\begin{aligned} P(\text{at least one distance} > 291.2) &= 1 - P(\text{all five distances} \le 291.2) \\ &= 1 - (1 - 0.1271)^5 \\ &= 1 - (0.8729)^5 \\ &= 1 - 0.5068 \\ &= 0.4932 \end{aligned}$$

Part (c):

Since the 99th percentile for a standard normal distribution is 2.33, we can set the appropriate z-score equal to 2.33 and solve for the desired mean, say M. Thus,

$\frac{291.2 - M}{2.8} = 2.33$ or $M = 291.2 - 2.33 \times 2.8 = 284.676$. In order to be 99 percent certain that a randomly selected ball does not exceed the maximum distance of 291.2 yards, the mean should be set to 284.676 yards.

3. Charity Fundraiser (2012 #2)

Part (a):

By counting the number of sectors for each value and dividing by 10, the probability distribution is calculated to be:

x	\$2	\$1	–\$8
$P(x)$	0.6	0.3	0.1

Part (b):

The expected value of the net contribution for one play of the game is:
$E(x) = \$2(0.6) + \$1(0.3) + (-\$8)(0.1) = \$0.70$ (or 70 cents).

Part (c):

The expected contribution after n plays is $\$0.70n$. Setting this to be at least \$500 and solving for n gives:

$0.70n \ge 500$, so $n \ge \frac{500}{0.70} \approx 714.286$,
so 715 plays are needed for the expected contribution to be at least \$500.

Part (d):

The normal approximation is appropriate because the very large sample size ($n = 1{,}000$) ensures that the central limit theorem holds. Therefore, the sample mean of the contributions from 1,000 plays has an approximately normal distribution, and so the sum of the contributions from 1,000 plays also has an approximately normal distribution.

The z-score is $\frac{500 - 700}{92.79} \approx -2.155$.

The probability that a standard normal random variable exceeds this z-score of -2.155 is 0.9844. Therefore, the charity can be very confident about gaining a net contribution of at least \$500 from 1,000 plays of the game.

4. Concert Tickets (2005B #2)

Part (a):

The mean of C is $0 \times 0.4 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 = 1$.

The standard deviation of C is $\sqrt{(0-1)^2 \times 0.4 + (1-1)^2 \times 0.3 + (2-1)^2 \times 0.2 + (3-1)^2 \times 0.1} = 1$.

Part (b):

Let $T = C + A$, where A is the total number of adult tickets purchased by a single customer, denote the total number of tickets purchased by a single customer.

The mean of T is $\mu_T = \mu_C + \mu_A = 1 + 2 = 3$.

The standard deviation of T is $\sigma_T = \sqrt{\sigma_C^2 + \sigma_A^2} = \sqrt{1^2 + 1.2^2} = \sqrt{2.44} = 1.562$.

Part (c):

Let $M = 15 \times C + 25 \times A$ denote the total amount of money spent per purchase.

The mean of M is $\mu_M = 15\mu_C + 25\mu_A = 15 \times 1 + 25 \times 2 = \$65$.

The standard deviation of M is $\sigma_M = \sqrt{15^2\sigma_C^2 + 25^2\sigma_A^2} = \sqrt{225 \times 1^2 + 625 \times 1.2^2} = \sqrt{1125} = \$33.54$.

5. Earth Depth (2006 #3)

Part (a):

Since $M = D + E$ (a normal random variable plus a constant is a normal random variable), we know that M is normally distributed with a mean of 2 feet and a standard deviation of 1.5 feet. Thus,

$$P(M < 0) = P\left(Z < \frac{0-2}{1.5}\right) < P(Z < -1.33) = 0.0918 \text{, where } Z = \frac{M-\mu}{\sigma}.$$

Part (b):

$$\begin{aligned} P(\text{at least one measurement} < 0) &= 1 - P(\text{all three measurements} \geq 0) \\ &= 1 - (1 - 0.0918)^3 \\ &= 1 - (0.9082)^3 \\ &= 1 - 0.7491 \\ &= 0.2509 \end{aligned}$$

Part (c):

Let $\bar{X}$ denote the mean of three independent depth measurements taken at a point where the true depth is 2 feet. Since each measurement comes from a normal distribution, the distribution of $\bar{X}$ is normal with a mean of 2 feet and a standard deviation of $\frac{1.5}{\sqrt{3}} = 0.8660$ feet. Thus,

$$P(\bar{X} < 0) = P\left(Z < \frac{0-2}{\frac{1.5}{\sqrt{3}}}\right) < P(Z < -2.31) = 0.0104 \text{, where } Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}.$$

6. Runners (2002 #3)

Part (a):

For runner 3

$$P(time < 4.2) = P\left(z < \frac{4.2 - 4.5}{.14}\right) = P(z < -2.14) = 0.0162 \quad (\textit{from table})$$

OR

$$P(time < 4.2) = 0.0160622279 \quad (\textit{from}\ \text{Calculator})$$

It is possible but unlikely that runner 3 will run a mile in less than 4.2 minutes on the next race. Based on his running time distribution, we would expect that he would have times less than 4.2 minutes less than 2 times in 100 races in the long run.

OR

It is possible but unlikely that runner 3 will run a mile in less than 4.2 minutes on the next race because 4.2 is more than 2 standard deviations below the mean. Since the running time has a normal distribution, it is unlikely to be more than 2 standard deviations below the mean.

Part (b):

$$\mu_T = \mu_1 + \mu_2 + \mu_3 + \mu_4 = 4.9 + 4.7 + 4.5 + 4.8 = 18.9$$

The runners' times are independently distributed, therefore

$$\sigma_T^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 = (.15)^2 + (.16)^2 + (.14)^2 + (.15)^2 = 0.0902$$

$$\sigma_T = \sqrt{.0902} = 0.3003$$

Part (c):

$$P(team\ time < 18.4) = P\left(z < \frac{18.4 - 18.9}{.3003}\right) = P(z < -1.67) = 0.0475 \quad (\textit{from table})$$

OR

$$P(team\ time < 18.4) = 0.0479561904 \quad (\textit{from}\ \text{Calculator})$$