AP Statistics Chapter 6 Review

NAME________________________________

1. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs. The weights of such full cartons are approximately normally distributed with a mean of 840 grams and a standard deviation of 7.9 grams.

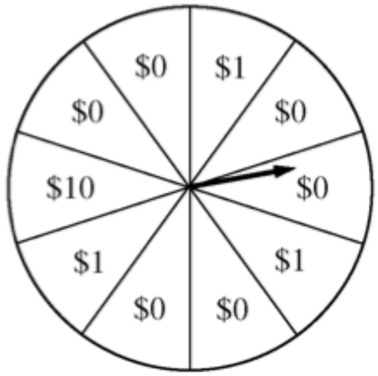
(a) What is the probability that a randomly selected full carton of Grade A eggs will weigh more than 850 grams?

(b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton. Let the random variable X be the weight of a single randomly selected Grade A egg.

i) What is the mean of X?

ii) What is the standard deviation of X?

2.
A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	–$8
$P(x)$			

(b) What is the expected value of the net contribution to the charity for one play of the game?

(c) The charity would like to receive a net contribution of $500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least $500 ?

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least $500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are $700 and $92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least $500 in 1,000 plays of the game.

ANSWERS:

1. 2013 #3 (Eggs)

Part (a):

Let W denote the weight of a randomly selected full carton of eggs. W has a normal distribution with mean 840 grams and standard deviation 7.9 grams.

The z-score for a weight of 850 grams is $z = \dfrac{850 - 840}{7.9} \approx 1.27$.

The standard normal probability table reveals that
$P(W > 850) = P(Z > 1.27) \approx 1 - 0.8980 = 0.1020.$

Part (b):

(i) Let W represent the weight of a randomly selected full carton of eggs, P the weight of the packaging, and X_i the weight of the *ith* egg, for $i =$ 1, 2, …, 12.

Note that $W = P + X_1 + X_2 + \ \ldots \ + X_{12}$.
Properties of expected values establish that $\mathrm{E}(W) = \mathrm{E}(P) + \mathrm{E}(X_1) \ + \ \ldots \ + \mathrm{E}(X_{12})$.
Because all 12 eggs have the same mean weight, this becomes $\mathrm{E}(W) = \mathrm{E}(P) + 12 \times \mathrm{E}(X_i)$.
We were told that $\mathrm{E}(W) = 840$ and $\mathrm{E}(P) = 20$, so we can solve
$840 = 20 + 12 \times \mathrm{E}(X_i)$ to find $\mathrm{E}(X_i) = \dfrac{840 - 20}{12} \approx 68.33$ grams.

(ii) Because of independence, properties of variance establish that
$$\mathrm{Var}(W) = \mathrm{Var}(P) + \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \ldots + \mathrm{Var}(X_{12}).$$

Because all 12 eggs have the same variance of their weights, this becomes
$$\mathrm{Var}(W) = \mathrm{Var}(P) + 12 \times \mathrm{Var}(X_i).$$

We were told that $\mathrm{SD}(W) = 7.9$ and $\mathrm{SD}(P) = 1.7$. Therefore, $\mathrm{Var}(W) = (7.9)^2 = 62.41$ and $\mathrm{Var}(P) = (1.7)^2 = 2.89$.
We can solve $62.41 = 2.89 + 12 \times \mathrm{Var}(X_i)$ to find $\mathrm{Var}(X_i) = \dfrac{62.41 - 2.89}{12} = 4.96$. Thus, $\mathrm{SD}(X_i) = \sqrt{(4.96)} \approx 2.23$ grams.

2. 2012 #2 (Charity Fundraiser)

Part (a):

By counting the number of sectors for each value and dividing by 10, the probability distribution is calculated to be:

x	\$2	\$1	–\$8
$P(x)$	0.6	0.3	0.1

Part (b):

The expected value of the net contribution for one play of the game is:

$E(x) = \$2(0.6) + \$1(0.3) + (-\$8)(0.1) = \$0.70$ (or 70 cents).

Part (c):

The expected contribution after n plays is $\$0.70n$. Setting this to be at least \$500 and solving for n gives:

$0.70n \geq 500$, so $n \geq \frac{500}{0.70} \approx 714.286$,

so 715 plays are needed for the expected contribution to be at least \$500.

Part (d):

The normal approximation is appropriate because the very large sample size $(n = 1{,}000)$ ensures that the central limit theorem holds. Therefore, the sample mean of the contributions from 1,000 plays has an approximately normal distribution, and so the sum of the contributions from 1,000 plays also has an approximately normal distribution.

The z-score is $\frac{500 - 700}{92.79} \approx -2.155$.

The probability that a standard normal random variable exceeds this z-score of -2.155 is 0.9844. Therefore, the charity can be very confident about gaining a net contribution of at least \$500 from 1,000 plays of the game.