AP Exam Review: Probability

3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

   After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

   Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

   (a) Calculate the expected score for each player.

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is -1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of -1, and calculate the probability for each combination.

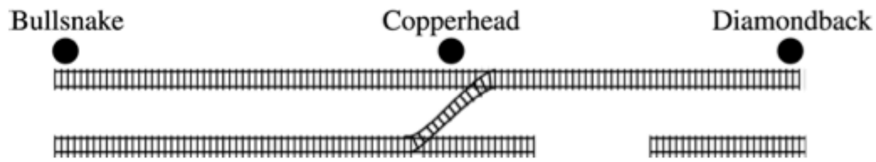
(c) Find the probability that the difference (Josephine minus Crystal) in their scores is -1.

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	-3	-2	-1	0	1	2
Probability	0.015			0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

5. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X, it takes the train leaving Bullsnake to get to Copperhead is normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y, it takes the train leaving Diamondback to get to Copperhead is normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

(a) What is the distribution of $Y - X$?

(b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?

(c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01 ?

5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.

   a. Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

   b. Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.

   (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

c	0	1	2	3
$p(c)$	0.4	0.3	0.2	0.1

(a) Compute the mean and the standard deviation of C.

(b) Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the numbers of child tickets and adult tickets purchased are independent random variables. Compute the mean and the standard deviation of the total number of adult and child tickets purchased by a single customer.

(c) Suppose each child ticket costs \$15 and each adult ticket costs \$25. Compute the mean and the standard deviation of the total amount spent per purchase.

ANSWERS: AP Exam Review: Probability

2004B #3: (Trains filled with ore)

Let X = weight of ore in a randomly selected car.

Part (a):

$$P(X > 70.7) = P\left(Z > \frac{70.7 - 70}{.9}\right) = P(Z > 0.78) = 0.2177$$

Part (b):

No. Approximately 22% of the cars will have ore weights of 70.7 or greater when the filling equipment is working properly, so a car that was filled with 70.7 tons of ore would not be an unusual occurrence.

Part (c):

$$P(\bar{X} > 70.7) = P\left(Z > \frac{70.7 - 70}{\frac{0.9}{\sqrt{10}}}\right) = P\left(Z > \frac{0.7}{0.285}\right) = P(Z > 2.46) = 0.0069$$

Part (d):

Yes, we would suspect that the filling mechanism is overfilling. If it is working properly, the probability that the mean weight of the ore in 10 randomly selected cars is 70.7 or greater is 0.0069, which is very small.

2008 #3 (Arcade Game)

Part (a):

The expected scores are as follows:

Josephine

$\mu_J = 16(0.1) + 17(0.3) + 18(0.4) + 19(0.2) = 17.7$

Crystal

$\mu_C = 17(0.45) + 18(0.4) + 19(0.15) = 17.7$

Part (b):

J	C	Probability
16	17	$(0.1)(0.45) = 0.045$
17	18	$(0.3)(0.40) = 0.12$
18	19	$(0.4)(0.15) = 0.06$

Part (c):

The probability is

$0.045 + 0.12 + 0.06 = 0.225$

Part (d):

P(difference = –1) = 0.225 (from part c)

P(difference = –2) = 1 – 0.015 – 0.225 – 0.325 – 0.260 – 0.90 = 0.085

Distribution of Josephine – Crystal

Differences	–3	–2	–1	0	1	2
Probability	0.015	**0.085**	**0.225**	0.325	0.260	0.090

The probability that Crystal's score is higher than Josephine's score is

P(difference < 0) = 0.015 + 0.085 + 0.225 = 0.325

1999 #5 (Die A and Die B)

Possible Outcomes

Die A	Die B	Winner	Prob
9	3	A	(2/3)(2/3) = 4/9
9	11	B	(2/3)(1/3) = 2/9
0	3	B	(1/3)(2/3) = 2/9
0	11	B	(1/3)(1/3) = 1/9

OR

	DIE A						
DIE B		0	0	9	9	9	9
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	11	B	B	B	B	B	B
	11	B	B	B	B	B	B

Winner	Prob
A	16/36 = 4/9
B	20/36 = 5/9

a. Choose die B, because the probability of winning is higher (5/9 compared to 4/9 for die A)

b. Let X be the number of tokens the player using die B should receive. For the game to be fair, we need

$45(4/9) = X(5/9)$

Solving this equation for X gives X = 36. Player B should receive 36 tokens.

2005 B #2 (Concert Tickets)

Solution

Part (a):

The mean of C is $0 \times 0.4 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 = 1$.
The standard deviation of C is $\sqrt{(0-1)^2 \times 0.4 + (1-1)^2 \times 0.3 + (2-1)^2 \times 0.2 + (3-1)^2 \times 0.1} = 1$.

Part (b):

Let $T = C + A$, where A is the total number of adult tickets purchased by a single customer, denote the total number of tickets purchased by a single customer.
The mean of T is $\mu_T = \mu_C + \mu_A = 1 + 2 = 3$.
The standard deviation of T is $\sigma_T = \sqrt{\sigma_C^2 + \sigma_A^2} = \sqrt{1^2 + 1.2^2} = \sqrt{2.44} = 1.562$.

Part (c):

Let $M = 15 \times C + 25 \times A$ denote the total amount of money spent per purchase.
The mean of M is $\mu_M = 15\mu_C + 25\mu_A = 15 \times 1 + 25 \times 2 = \$65$.
The standard deviation of M is $\sigma_M = \sqrt{15^2 \sigma_C^2 + 25^2 \sigma_A^2} = \sqrt{225 \times 1^2 + 625 \times 1.2^2} = \sqrt{1125} = \$33.54$.