# AP Exam Review: Sampling Distributions

## 2007 AP® STATISTICS FREE-RESPONSE QUESTIONS

3. Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

(a) Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely?

- A random sample of 15 fish having a mean length that is greater than 10 inches or
- A random sample of 50 fish having a mean length that is greater than 10 inches

Justify your answer.

(b) Suppose the standard deviation of the sampling distribution of the sample mean for random samples of size 50 is 0.3 inch. If the mean length of the fish is 8 inches, use the normal distribution to compute the probability that a random sample of 50 fish will have a mean length less than 7.5 inches.

2. A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.

   (a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

   (b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

2. A tire manufacturer designed a new tread pattern for its all-weather tires. Repeated tests were conducted on cars of approximately the same weight traveling at 60 miles per hour. The tests showed that the new tread pattern enables the cars to stop completely in an average distance of 125 feet with a standard deviation of 6.5 feet and that the stopping distances are approximately normally distributed.

   (a) What is the 70th percentile of the distribution of stopping distances?

   (b) What is the probability that at least 2 cars out of 5 randomly selected cars in the study will stop in a distance that is greater than the distance calculated in part (a) ?

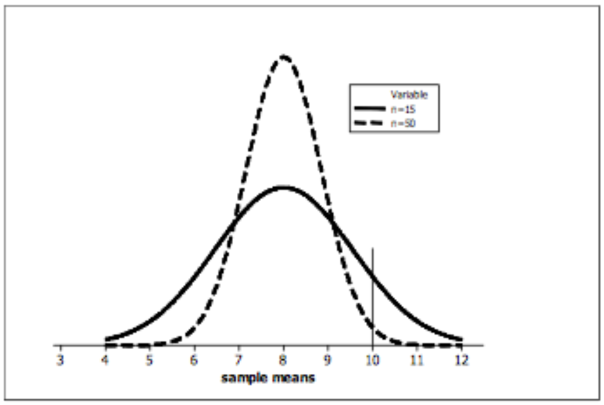
   (c) What is the probability that a randomly selected sample of 5 cars in the study will have a mean stopping distance of at least 130 feet?

# ***ANSWERS:*** AP Statistics Exam FR: Sampling Distributions

2007 #3 (Big Town Fisheries)

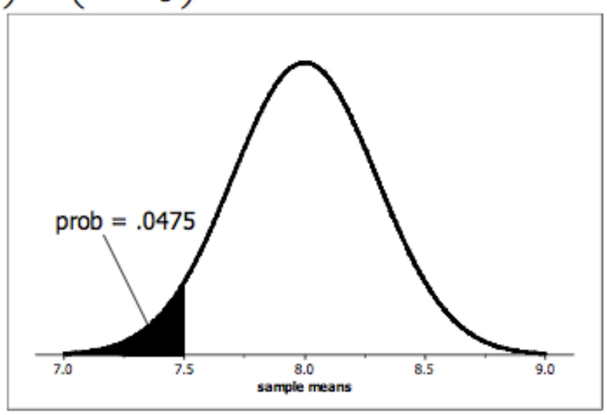
**Part (a):**

The random sample of $n = 15$ fish is more likely to have a sample mean length greater than 10 inches. The sampling distribution of the sample mean $\bar{x}$ is normal with mean $\mu = 8$ and standard deviation $\sigma/\sqrt{n}$. Thus, both sampling distributions will be centered at 8 inches, but the sampling distribution of the sample mean when $n = 15$ will have more variability than the sampling distribution of the sample mean when $n = 50$. The tail area ( $\bar{x} > 10$) will be larger for the distribution that is less concentrated about the mean of 8 inches when the sample size is $n = 15$, as shown in the following graph.




**Part (b):**

$$P(\bar{x} < 7.5) = P\left(z < \frac{7.5 - 8}{0.3}\right) = P\left(z < -\frac{5}{3}\right) = P(z < -1.67) = 0.0475$$




**Part (c):**

Yes. The Central Limit Theorem says that the sampling distribution of the sample mean will become approximately normal as the sample size $n$ increases. Since the sample size is reasonably large ($n = 50$), the calculation in part (b) will provide a good approximation to the probability of interest even though the population is nonnormal.

2010 #2 (Radio Station)

**Part (a):**

The sampling distribution of the sample mean song length has mean $\mu_{\bar{X}} = \mu = 3.9$ minutes and standard deviation $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.1}{\sqrt{40}} \approx 0.174$ minutes. The central limit theorem (CLT) applies in this case because the sample size ( $n = 40$ ) is fairly large, especially with the population of song lengths having a roughly symmetric distribution. Thus, the sampling distribution of the sample mean song length is approximately normal.

**Part (b):**

The probability that the total airtime of 40 randomly selected songs exceeds the available time (that is, the probability that the total airtime of 40 randomly selected songs is greater than 160 minutes) is equivalent to the probability that the sample mean length of the 40 songs is greater than $\frac{160}{40} = 4.0$ minutes.

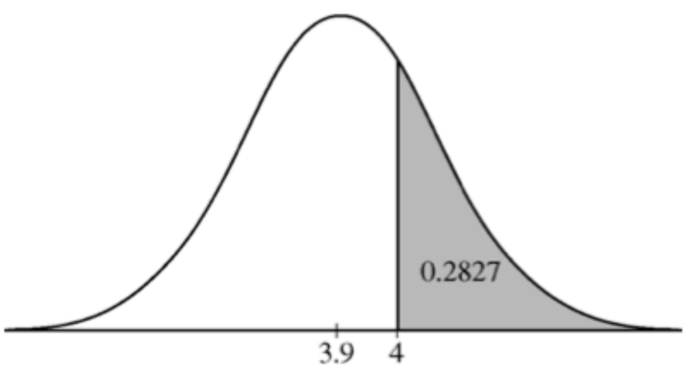
According to part (a), the distribution of the sample mean length $\bar{X}$ is approximately normal. Therefore,

$$P\left(\bar{X} > 4.0\right) \approx P\left(Z > \frac{4.0 - 3.9}{0.174}\right) = P(Z > 0.57) = 1 - 0.7157 = 0.2843.$$

(The calculator gives the answer as 0.2827.)

The approximate sampling distribution of the sample mean song length and the desired probability are displayed below.

Mean = 3.9, StDev = 0.174




Sample Mean Song Length

**Part (b) (*alternative*):**

An equivalent approach is to note that the sampling distribution of the total airtime, $T$, for the 40 songs is approximately normal, with mean $40(3.9) = 156$ minutes and standard deviation $\sqrt{40}(1.1) \approx 6.96$ minutes. The $z$-score for a total airtime of 160 minutes is then $z = \frac{160 - 156}{6.96} \approx 0.57$, and the calculation proceeds as above.

2009 #2 (Tire Manufacturer)

**Part (a):**

Let $X$ denote the stopping distance of a car with new tread tires where $X$ is normally distributed with a mean of 125 feet and a standard deviation of 6.5 feet. The $z$-score corresponding to a cumulative probability of 70 percent is $z = 0.52$. Thus, the 70th percentile value can be computed as:

$$x = \mu_X + z\sigma_X = 125 + 0.52(6.5) = 128.4 \text{ feet.}$$

**Part (b):**

From part (a), it was found that a stopping distance of 128.4 feet has a cumulative probability of 0.70. Thus the probability of a stopping distance greater than 128.4 is $1 - 0.70 = 0.30$.

Let $Y$ denote the number of cars with the new tread pattern out of five cars that stop in a distance greater than 128.4 feet. $Y$ is a binomial random variable with $n = 5$ and $p = 0.30$.

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \left[\binom{5}{0}(0.3)^0(0.7)^5 + \binom{5}{1}(0.3)^1(0.7)^4\right]$$
$$= 1 - 0.5282 = 0.4718.$$

**Part (c):**

Let $\bar{X}$ denote the mean of the stopping distances of five randomly selected cars. All tires have the new tread pattern. Because the stopping distance for each of the five cars has a normal distribution, the distribution of $\bar{X}$ is normal with a mean of 125 feet and a standard deviation of $\frac{6.5}{\sqrt{5}} = 2.91$ feet. Thus,

$$P(\bar{X} > 130) = P\left(Z > \frac{130 - 125}{6.5/\sqrt{5}}\right) \approx P(Z > 1.72) = 0.0427.$$